THEORETICAL AND EXPERIMENTAL DYNAMIC ANALYSIS OF RAKANJI STONE ARCH BRIDGE, HONYABAKEI, OITA, JAPAN

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Abstract

Rakanji stone arch bridge built in 1920 over Yamakuni River, Honyabakei, Oita, has three spans of about 26m and total length of the bridge is about 89.3m. For the purpose of obtaining the data concerning dynamic structural properties of Rakanji stone arch bridge, both microtremor measurement by ambient vibrations and acceleration by traffic vibrations are measured. From the material tests, Young’s modulus and compressive strength of the stone and the mortar are estimated to be 17000MPa, 5500MPa – 7800MPa, 28.5MPa and 4MPa – 9MPa, respectively. From the results of dynamic tests, the fundamental frequencies of Rakanji stone arch bridge are estimated to be about 5.3Hz and 7.6Hz in out-of-plane and vertical directions, respectively. Its natural modes are identified by ARMAV (Auto Regressive Moving Average) and ERA (Eigensystem Realization Algorithm) models comparing with the results obtained by FEA (Finite Element analysis).

Introduction

Stone and/or brick are usually used as construction materials in Europe from thousands years ago. There are a great number of masonry structures in Europe. Also in Japan, especially in Kyushu, there are over a thousand stone arch bridges built before 1925. The arch technology perfected in ancient Rome was introduced to Nagasaki by the Portuguese in the early 17th century. The oldest remaining stone arch bridge in Japan, “Nagasaki Spectacles Bridge” was constructed in 1634. The name is derived from the appearance of the bridge and its reflection in the river. Rakanji stone arch bridge is also one of the most important stone arch bridges in Japan (Nagasaki web site). The purpose of this paper is to obtain the data concerning dynamic structural properties such as natural frequencies and modes of Rakanji stone arch bridge.

Rakanji Stone Arch Bridge, Honyabakei

The construction of Rakanji stone arch bridge over Yamakuni River, Honyabakei, Oita, was started in 1917 (Figure 1). During the construction, stone arch bridge was fallen down for two times and it was completed in 1920. It has three spans of about 26m and total length of the bridge is about 89.3m (Figure 2). The rise of the arches is about 5.1m
and the radical thickness of the stone arches is about 0.93m. Rise span ratio is about 0.19, in other word, arches are very shallow. The width of the bridge is about 4.5m (Honyabakei web site).

(a) View from down stream side  
(b) View from bank

Figure 1. Rakanji stone arch bridge over Yamakuni River, Honyabakei, Oita, in Japan.

Material Test

In order to estimate Young’s modulus and compressive strength of Rakanji stone arch bridge, core-drilling test are carried out (Figure 3). The diameter and height of the specimens are 24mm and 48mm, respectively. From the material tests, as for the stone of the spandrel wall, specific gravity in air-dry condition, compressive strength, and Young’s modulus are estimated to be 1.83, 28.5MPa, and 17224.2MPa, respectively. As for the mortar, compressive strength and Young’s modulus are determined as 4MPa – 9MPa and 5500MPa – 7800MPa, respectively (Constec, 2003).
As for the fill material, endoscope and overcorning test are carried out. From the top to the ground level, fill material consists of asphalt, deck slab, tuff-braccia, clay, conglomeratic, rudaceous, welded tuff, tuffaceous andesite, river gravel, and cavity (Constec, 2003).

![Stone](image1)
![Mortar](image2)

(a) Stone  (b) Mortar

Figure 3. Core sampling specimens.

Dynamic Tests

In order to estimate dynamic structural properties such as natural frequencies and modes, both microtremor measurement by ambient vibrations and acceleration by traffic vibrations are measured.

Microtremor Measurement

For the purpose of obtaining the data concerning dynamic structural properties of Rakanji stone arch bridge, as a first phase of dynamic test, microtremor by ambient vibrations are measured at different points shown in Figure 2. Smoothed spectra by Parzen’s spectral window of 0.1Hz are given in Figure 4. In Figure 4, fine solid line, bold dotted line and bold solid line show the spectra of longitudinal, out-of-plane and vertical directions, respectively. According to the observation of microtremor measurement, the fundamental frequencies of Rakanji stone arch bridge are estimated to be 5.3Hz and 7.6Hz in out-of-plane and vertical directions, respectively.

![Figure 4](image3)

(a) Point 11 in Figure 2  (b) Point 14 in Figure 2

Figure 4. Spectra observed in Microtremors.
**Acceleration Measurement**

In the second phase of dynamic test, acceleration of 6 points is contemporaneously measured in vertical direction. 5 setups of 6 measuring points are prepared as shown in Figure 2 and Table 1. Two sensors are fixed at both points 2 and 12 in Figure 2 and others are placed according to Table 1. Excitation is ground motion due to truck.

**Table 1. Measurement setup for Rakanji stone arch bridge.**

<table>
<thead>
<tr>
<th>Setup</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>12</td>
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<tr>
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<td></td>
<td>2</td>
<td>12</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

**Dynamic Identification**

The experimental dynamic parameters such as fundamental frequencies, modal shapes and damping factors are identified by analysis of the accelerations time-history by means of ARMAV (Auto Regressive Moving Average) and ERA (Eigensystem Realization Algorithm) techniques.

**The ARMAV Technique**

An ARMAV model (Marple, 1987) can be expressed in the state space according to the following expression (u and x are the input and output):

\[
\begin{bmatrix} \{x[n]\} \\
\{u[n]\}
\end{bmatrix} = \begin{bmatrix} a \end{bmatrix} \begin{bmatrix} x[n-1]\end{bmatrix} + \begin{bmatrix} b \end{bmatrix} \{u[n]\}
\]

(1)

In these parametric models the system output \( \vec{x}[n] \) is supposed to be caused by a white noise input \( \vec{u}[n] \) and the algorithm estimates the parameters’ values that minimize the residual variance. The parameter estimation algorithm works as follows: a first ARV model, whose structure is

\[
\vec{x}[n] = \sum_{k=1}^{p} \hat{A}[k] \vec{x}[n-k] + \hat{u}[n]
\]

(2)

is fitted to the data. Using the estimated autoregressive parameters \( \hat{A}[k] \), the residual vector \( \hat{u}[n] \) is computed and used as input for the ARMAV model:
\[
\bar{x}[n] = \sum_{k=1}^{p} \hat{A}[k] \bar{x}[n-k] + \hat{u}[n] + \sum_{k=1}^{q} \hat{B}[k] \bar{y}[n-k]
\] (3)

An iterative procedure can then be started, to alternately refine the estimated parameters \( \hat{A}[k] \), \( \hat{B}[k] \) and the residual \( \hat{u}[n] \) to minimize the residual variance. The procedure ends when the difference between the parameters \( \hat{A}[k] \) and \( \hat{B}[k] \), estimated in two consecutive iterations, is smaller than a desired value.

*The ERA Technique*

An Era model (Juang and Pappa, 1984) allows determining the modal parameters such as frequencies, damping factors and modal shapes of a dynamic system based on the description of its behavior in the space of the phases.

The general principle is to tie the theoretical response to the measured response in the points of acquisition. Treating of discrete process, the relationship is:

\[
\{x(k)\} = [R]\{u(k)\} = [R][A]^{-1}\{B\}
\] (4)

where \([A]\) and \([B]\) are the model parameters matrices and \([R]\) is a transformation matrix.

Considering the excitement of all the points of input can be written:

\[
[X(k)] = [R][A]^{-1}\{B\}
\] (5)

The matrices \([X(k)]\), called Markov parameters (response functions to the impulse), are used for the construction of the matrices of generalized Hankel:

\[
[H(k-1)] = \begin{bmatrix}
[X(k)] & [X(k+1)] & \cdots & [X(k+j)] \\
[X(k+1)] & [X(k+2)] & \cdots & [X(k+j+1)] \\
\vdots & \vdots & \ddots & \vdots \\
[X(k+i)] & [X(k+i+1)] & \cdots & [X(k+i+j)]
\end{bmatrix}
\] (6)

Through the use of the Singular Value Decomposition is possible to determine the minimum order of the system and to trace back again the modal parameters of the model such as frequencies, damping factors and modal shapes.

**Finite Element Model**

The finite element model is composed of 20-node isoparametric solid elements (Figure 5). From the material tests, Young’s moduli of the stone and the mortar are estimated to be 17224MPa and 5500MPa – 7800MPa, respectively. As for Young’s modulus of arch,
spandrel wall and pier, depend on the thickness of the mortar, are expected the same and/or larger than that of mortar, but less than that of the stone. In this analysis, Young’s modulus of them is assumed to be 7800MPa. On the other hand, Young’s modulus of fill material is expected less than that of mortar. Its Young’s modulus is assumed to be here 1/10 of that of the mortar, that is 780MPa. Specific gravity is assumed to be 1.83. Total number of nodes and elements are 24493 and 4552, respectively. The boundary condition of two piers and both sides is assumed to be fixed.

Figure 5. Analytical model.

Figure 6. Frequency distribution identified by ARMAV model.

Figure 7. Frequency distribution identified by ERA model.
Results of Dynamic Identification

In Figures 6 and 7, the frequency distributions of Rakanji stone arch bridge identified by the ARMAV and ERA models are depicted in vertical directions, considering the complete time record, respectively. Fundamental frequencies and modes in vertical direction identified by the ARMAV and ERA models are listed in Table 2 comparing with the results of numerical analysis using Finite Element Method.

Table 2. Natural frequencies and modal shapes identified by ARMAV and ERA models comparing with those determined by FEA.

<table>
<thead>
<tr>
<th>Theoretical</th>
<th>Experimental</th>
<th>ARMAV</th>
<th>ERA</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODE=4</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>8.03Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MODE=5</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.39Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.25Hz</td>
<td>9.18Hz</td>
<td></td>
</tr>
</tbody>
</table>

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Concluding Remarks

The following concluding remarks were obtained.

1. From the material tests, Young’s modulus and compressive strength of the stone and the mortar are estimated to be 17000MPa, 5500MPa – 7800MPa, 28.5MPa and 4MPa – 9MPa, respectively.

2. From the results of dynamic tests, the fundamental frequencies of Rakanji stone arch bridge are estimated to be about 5.3Hz and 7.6Hz in out-of-plane and vertical directions, respectively.

3. Its natural modes are identified by ARMAV (Auto Regressive Moving Average) and ERA (Eigensystem Realization Algorithm) models comparing with the results obtained by FEA (Finite Element analysis).

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