Structural Analysis with F.E. method of the elliptical dome of the Sanctuary of Vicoforte

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Abstract

The Sanctuary of Vicoforte, near Mondovì (Cuneo, Italy), is a building of great historical-architectural significance. It began to be constructed in 1594 by architect Ascanio Vittozzi from Orvieto. Declared a national monument in 1918, the Sanctuary owes it fame primarily to its great elliptical dome (figs. 1-2), erected in 1732, which is the fourth biggest in the world (after Saint Peter, S. Maria del Fiore, the Gol Gumbaz Mausoleum in India) and, with its 37.15 m and 24.80 m long axes, is by far the largest elliptical dome the world over. Since the earliest stages of construction, the building was adversely affected by settlements of the foundations, due to the conditions of the soil. These problems slowed down the building process and at some point even aroused fears for its collapse. The structure still reveals a complex network of cracks that affects most of building and appears to be particularly severe in the dome-drum system (fig. 3). Over the past decades, the building has been kept under observation and strengthened; in 1985 active cerclage bars were applied to the top of drum, and a complex monitoring system was set up to measure the movements of structures, especially in the cracked portions of the dome. The finite element analysis described in this study was conducted within the framework of a broad program of improvement of the monitoring system. The investigations underway, whose results will be entered in the numerical model, include: topographic survey, geotechnical and hydro-geological

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investigations, tests on the materials, etc... The model will be progressively refined to obtain a tool providing useful information on the current status of the structure, the efficacy of the old annular iron rings and the modern strengthening system, the likely effects on the buildings of catastrophic events or further strengthening works, the general safety level of the structure. This study illustrates the results of the analyses conducted with the numerical model in its initial stage of development. So far, significant results have been obtained especially as concerns the

interpretation of the static behaviour of the architectural structures.

Figure 1. (a) General view of the Sanctuary of Vicoforte (Mondovì, Italy). (b) Internal view of the dome

1. Brief historical notes

The origins of the Sanctuary of Vicoforte are associated with the devotion of the local population for *Nostra Signora Montis Regalis*: in 1594, after a miraculous event, the inhabitants of Vicoforte decided to erect a Sanctuary, precisely where the miracle had occurred. The great devotion of the people and the stream of pilgrim visiting the Sanctuary aroused the interest of Duke Charles Emanuel I of Savoy, who, in 1595, decided to interrupt the works underway and build a more elaborate church, that might serve as a mausoleum for the Savoy dynasty.

The construction of the new Sanctuary began in 1596 at the hands of architect Ascanio Vittozzi from Orvieto (1539-1615); the chosen model can be traced back, from the liturgical viewpoint, to the rules set forth by the Council of Trento (1545-1563), and, at the symbolic level, to the Pantheon and the early roman churches with oval plan (e.g. Saint James in Augusta in Rome by F. da Volterra, 1592). Vittozzi's Sanctuary was designed to be large enough to accommodate in its central part the previous church, that was preserved until 1725 and used as a chapel for liturgical functions until the completion of the new building. The choice of the site, prompted by religious considerations, turned out to be unfortunate from the very start, due to the characteristics of the soil: the building, in fact, rests on a variable layer of clay followed by a marl layer, from 3 to 9 m deep, declining in the south-west direction (fig. 4). Important settlements developed during the first phases of the construction. Moreover, the dome-drum system designed by Vittozzi seemed, at first, not feasible from the static viewpoint. This explains why works were halted in 1600, when the construction of the church had gotten to the level of the impost of the big arches giving access to the chapels (10 m above ground) and why Vittozzi dug drainage channels under the foundations of the Sanctuary.

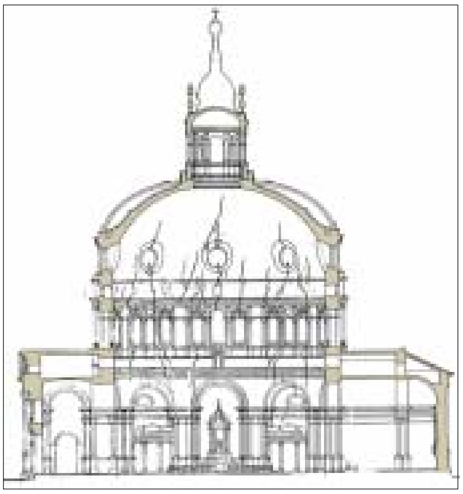

Figure 2. (a)Cracks in the dome. North-south section; view of the west side (Garro 1962). (b) Axonometric view of foundation and foundation layers

For approx. one century, the building site remained virtually inactive until, in 1701, works were resumed under the direction of young architect Francesco Gallo (1672-1750). With the construction of the dome in 1732 and the lantern the following year, the Sanctuary of Vicoforte was virtually completed.

In the course of the following centuries, concerns about new settlements of the foundations and the opening of a vast network of cracks (fig.3), in particular in the dome and the drum, prompted several investigation and strengthening campaigns. The dome had been equipped by Gallo by an annular strengthening system consisting of 3 iron rings embedded in the masonry and situated in the proximity of the oval windows at the level of the outer impost, as shown in figure 6, (for a total of 140 cm^2 of resisting section). The strengthening system applied by Gallo was deemed insufficient to ensure the stability of the dome and hence, in 1987, a new strengthening system was put in place.

The modern strengthening system consists of 14 groups of tangential ties distributed along the perimeter of the drum, each group comprising 4 superimposed ∅32 mm Dywidag bars (32 cm^2 in all) of high-strength steel, each bar being located in ducts drilled into the masonry. The 14 groups of bars are interconnected by steel frames. The force in the tie-bars may be regulated at any time by means of jacks contracting on the head frames and stress levels are constantly monitored by load cells (fig.6). As demonstrated below by model analysis the position of the original strengthening system was the most appropriate. The same analysis proves that the position of the modern strengthening system, located for practical reasons in the upper part of the drum, is also adequate.

2. Analysis of the geometry of the Vicoforte Sanctuary.

The church of *N.S. Montis Regalis* of Vicoforte was erected to serve a twofold function: a sanctuary and a mausoleum for the Savoy family. To this end, the building was designed with a central plan, having at its centre the image of the Virgin with Child, to whom the Sanctuary was dedicated, and the chapels, that were meant to host the tombs of the dukes of Savoy, arranged radially all around. Since according to the rules defined by the Council of Trento all churches were to have a dominant longitudinal axis, Vittozzi selected an oval as the basic shape of the building. As borne out by the treatises of the time, the correct method to draw an ellipse was not known yet, and even at a later time vaults of this type were built on an oval, since this geometric shape made it easier to build the centring. The dome is much bigger than in the churches designed till then. If we compare it with S. James in Augusta, the largest church of the same period and type, we find an appreciable difference in scale. The fact that the Sanctuary of Vicoforte was meant to be majestic to attest the magnificence of the Savoy dynasty does not justify the risk taken in building a dome of this size: the main reason was that the central area of the Sanctuary had to be big enough to accommodate the pre-existing church in order to preserve a place of worship for the pilgrims that flocked to Vicoforte (fig.5).

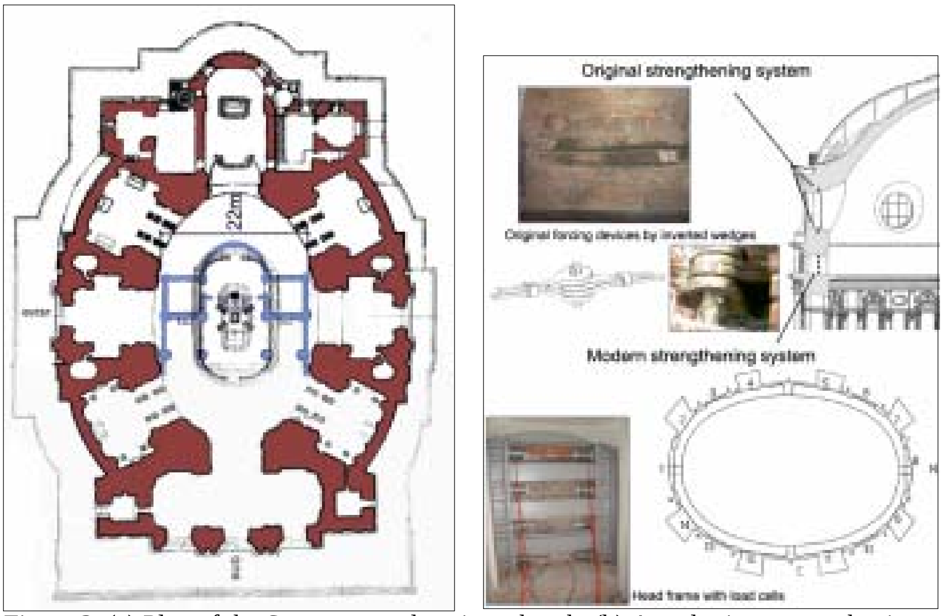

Figure 3. (a) Plan of the Sanctuary and ancient church, (b) Annular iron strengthening applied by Gallo (1732) and modern strengthening system (1987)

Vittozzi’s project for the dome-drum system was forsaken on account of its being structurally unfeasible: it envisaged a renaissance styled dome, designed like the hemispherical domes with the outer impost at the same level as the inner impost.

The new design of the dome by Gallo comformed to the characters of baroque architecture and incorporated the experience developed in the construction of the large domes in the preceding century. As borne out by architectural practice, in ellipsoidal domes it is necessary to increase the height of the outer impost in order to prolong the extrados of the drum and hence transfer a bigger vertical load onto the piers and centre the large radial thrusts, as well as make the buttresses as tall as possible. Another aspect to be underscored is the special profile of the extrados of the dome of the Sanctuary, which, at mid height of the arch has a step (approx. 65 cm wide) and at the bottom has a reversed arch giving rise to a marked thickening of the dome in the proximity of the piers. The sectional view of the dome resembles the ideal profile of hemispheric domes advocated by Abbot Mascheroni on the basis of mathematical considerations in his work *Nuove ricerche sull’equilibrio delle volte (New investigations into the equilibrium of vaults*), published in 1785. The thickness of the vault varies from 230 cm at the impost to 117/134 cm at the base of the lantern. The vault is strengthened by 8 ribs (one for each buttress).

3. Innovative restoring and monitoring technologies

At present, the Sanctuary of Vicoforte is equipped with a monitoring system installed in 1985. In addition to load cells at the tie bars, it uses a large number of displacement measuring devices (partly mechanical and partly electrical and automatic) applied to the main cracks, as well as temperature gauges. Measures of variations in diameter have been performed manually so far. At present the monitoring system is being completed. A new five-year program of monitoring, research and investigations - referred to as *Vicoforte 2002-2006* - designed to control the present conditions of structural conservation of the monument and to define correct criteria for its future maintenance and, where needed, further structural restoration, was started in January 2002 under the coordination of Politecnico di Torino. As a part of this program, a joint research project has been established between Politecnico di Torino and a group of Japanese Universities and research institutions.

The goals of the Vicoforte 2002-2006 program include:

l. Improvement of the geometrical and structural description

2. Diagnostic inspection of deterioration and structural damage by means of non-destructive tests

3. Characterization of the properties of the masonry through non-destructive and, where feasible, partially destructive tests

4. Investigation into the mechanical characteristics and continuity of the three original sets of annular iron ties embedded at the base of the dome

5. Geotechnical characterization of the foundation layers through field and laboratory tests

6. Installation of a new and extended monitoring system for the continuous control of the main parameters characterizing the behaviour of the monument, with special regard to the dome, and the foundation layers

7. Measurements of ambient vibration modes and natural frequencies and continuous recording of the dynamic responses to seismic events of specified magnitude

8. Interpretation of the static and dynamic behaviour of the dome and of the monument as a whole (including its foundations layers) by means of finite element three-dimensional elasto-plastic and dynamic analyses, based on appropriate damage criteria

9. Updating the numerical models by comparison with experimental data

10. Estimation of the drum-dome global safety level through limit analysis.

11. Proposals for structural conservation and maintenance with special regard to the use of post-tensioning rings at the base of the dome and the definition of optimal stress levels for the steel tie-rods

12. Analysis of the site from the hydrological point of view and estimation of the risks associated with extreme events.

The numerical model

This numerical model is the fruit of an investigation undertaken about a decade ago with a study regarding the dome, which now has been extended to the geometry of the entire Sanctuary. The plan is to proceed with a progressive refinement of the model so as to obtain a tool designed to conduct investigations in different fields of analysis. The study presented herein is the first stage of the analysis that, albeit characterised by a simple formulation, has already yielded significant findings. The model produced is of linear-elastic type and was created in three-dimensional form by means of computation code Ansys(R) 6.0. It uses 10158 20-node brick elements (solid95) and 580 8-node shell elements (shell93) to model the vaults of the chapels, to a total of 192864 degrees of freedom. At this stage, the only load considered is dead load, and the foundation layer supporting the monument is considered rigid. The other loads (e. g. snow or wind action) have been disregarded; also the effect of the old and new strengthening systems and the effect of foundation settlements have not been considered, as well as the lack of tensile strength of the masonry. Notwithstanding these limitations, the analysis represents an essential step in our investigation, since it helps us gain an understanding of the potential of this tool and supplies significant data on the structural implications of architectural forms.

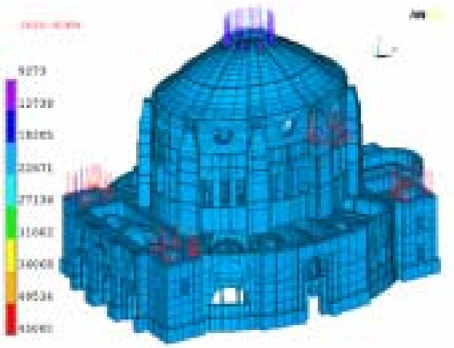

Figure 4. Numerical model

Analysis of the results

Since the main goal of the analysis was to be able to interpret the static behaviour of the Sanctuary, the attention was mainly focused on graphic outputs. Node displacement data supply an image of the way the geometry of structure deforms, revealing the way the different architectural elements work. On the other hand, stress analysis by means of the graphic images makes it possible to identify the distribution of stresses in the building, with special regard to the zones where their values are highest or critical.

Through an analysis of the principal stresses, it proved possible to identify the zones subject to the most adverse conditions, i.e., the zones where the principal compressive stresses reach the highest values and tensile stresses are generated.

To gain a better understanding of the structural behaviour of the Sanctuary of Vicoforte, the solutions supplied by the complete model have been compared with those obtained from partial models, to identify the interactions between the different building components.

The model has shown that the entire structure weighs approximately 77000 t, of which 16500 t is accounted for by the weight of the dome-drum system, 500 t by the lantern and 1700 t by each bell tower. Maximum compressive stresses, of 2.4 N/mm^2, are situated at the base of the buttresses (fig.8). This is justified by two factors:

• the loads of the dome and drum at this level are distributed over a smaller section: at lower levels, the loads increase but the resisting section increases to an appreciable extent.

• compressive stresses are concentrated on the outer profile of the buttresses due to the rotation of the drum section.

Failure stresses determined on samples of the masonry extracted from structure in the same regions are situated in the range of 2÷2.5 N/mm^2 for one of the buttresses (2 samples). Two other samples extracted from different buttresses show a failure stress of 4.7 and 5.8 N/mm^2, respectively. The concern about the high values of the compressive stresses determined on the model, however, should take into account the fact that these values are extreme values extended on limited portions of the resisting section only.

At the base of the construction, maximum compressive values, of 1.1 N/mm^2, are observed at the two pillars on the southern side, while in the dome the highest compressive stress (1.4 N/mm^2) is observed at the level of the keystone ring, due to the effects of the load of the lantern.

The main structural problems of the Sanctuary of Vicoforte certainly stem from the tensile stresses that occur at different points of the building and are mostly concentrated in the dome impost zone and in a vast area of the drum immediately below. In actual fact it should be considered that the maximum tensile values determined by f.e.m. analysis are fictitious, since, as mentioned above, masonry is not a tensile resistant material. At all events, the tensile stress zones correspond well to the regions identified as the origin of cracking processes. Broad zones are seen to be affected by principal tensile stresses, especially in the tangential direction, in many parts of the structure. It should be noted, however, that these values are close to zero, i.e., too small to cause cracking. The maximum tensile value, of ca 0.61 N/mm^2, is observed at the top of the oval windows situated over the smaller diameter of the dome, a region characterized by severe cracking, confirming the fact that these openings represent the weakest point in the entire structure (figs.9-10).

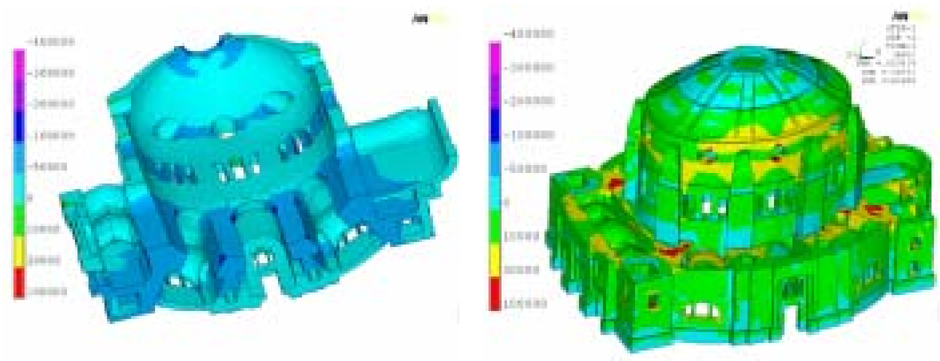

Figure 5. (a) Diagram of principal (sub-vertical) stresses S3. (b) Diagram of principal (sub-horizontal) stresses S1

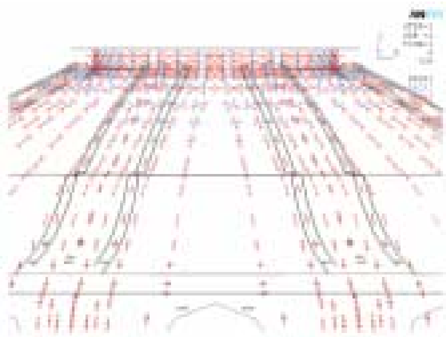

Figure 6. Vectorial representation of principal stresses: detail of the dome.

From an analysis of the way model geometry deforms, it is possible to identify the causes of the cracking phenomena affecting the building and hence formulate a hypothesis on the structural behaviour of the Sanctuary, by comparing the results and correlating them with the cracking configuration.

An analysis of the principal stresses in the dome shows how those running in the sub-vertical direction are mainly compressive and turn out to be higher where the masonry is thicker. This situation is beneficial to the stability of the dome, as the zones where sub-vertical compressive stresses are highest help to stiffen the entire dome with respect to the tensile stresses arranged in the tangential direction. This proves the importance of the presence of the ribbing at the extrados.

Horizontal stresses tangent to the perimeter of the dome are compressive in the upper part, whilst at the outer impost of the dome they turn into tensile stresses. The highest values are observed at the oval windows located along the smaller diameter of the elliptical vault. Tensile stresses tangent to the perimeter of the dome arise from the horizontal actions associated with the weight of the dome. These thrusts are stronger in the portions of the dome where the radius of curvature is smaller (figs.11-12).

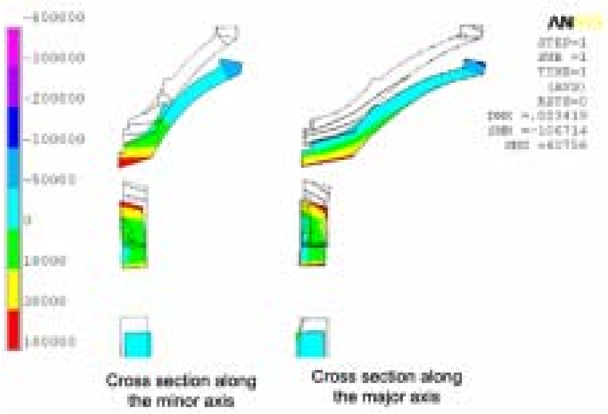

Figure 7. Diagram of the stresses normal to the sections along the minor and the major axis of the dome.

This phenomenon is also attested by the diagrams depicting the horizontal translation components of the dome elements; their intensity is particularly appreciable along the east-west axis and at the level of the oval windows (figs. 13-14). The horizontal actions are also present along the bigger diameter, but their intensity is not as strong. The horizontal actions of the dome also affect the drum, where they cause a rotation of the masonry section, according to different modalities depending on the zone, as clearly borne out by the vectorial representations of the displacements. In the masonry sections of the drum, along the shorter axis, the upper part of the drum is pushed outwards, while the base of the drum converges inward. A different, slightly asymmetrical behaviour is observed along the longer axis, due to the different configuration of the underlying masonry. On the northern side, the drum does not undergo an appreciable rotation, but on the southern side the space of the atrium works like a yielding restraint and the masonry section of the drum undergoes a rotation in a direction opposite to that of the other axis, albeit much less noticeable. The deformation imposed on the drum also affects the configuration of the sub-vertical stresses in the buttresses. At the base of the buttresses adjacent to the smaller diameter we find the highest compressive stresses arranged in the vertical direction. This is due to the rotation of the buttresses and the ensuing concentration of the compressive stresses on the exterior of the reacting section, which, however, does not crack on account of it being totally compressed. Though the vertical loads due to the dead load transferred by the buttresses to the columns underneath increase, the stresses at the base of the Sanctuary decrease in intensity, owing to an appreciable increase of the reacting section.

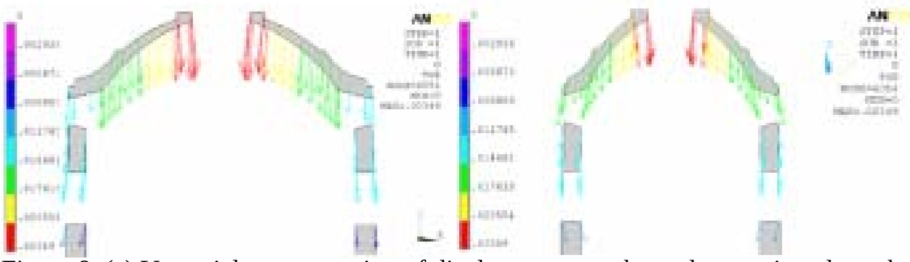

Figure 8. (a) Vectorial representation of displacements at the nodes: section along the major axis of the dome. (b) Vectorial representation of displacements at the nodes: section along the minor axis of the dome.

4. Concluding remarks

The strain and stress configuration reproduced by the numerical model matches quite closely the cracking pattern observed in the Sanctuary of Vicoforte. Compressive stresses are sufficiently lower than failure stresses and therefore it can be stated that the cracking conditions of the dome are mostly due to the tensile stresses (basically those running horizontally), the settlements of the foundations and thermal phenomena. Though these factors overlap, it proves possible to identify in the cracking configuration a close correspondence with the state of stress described by the model. In particular, analogies are observed between real conditions and the conditions reproduced by the model if we compare the positions of the cracks in the masonry with those in the zones affected by tensile stresses. This is exemplified most clearly by the thick network of cracks located between the tripartite windows of the drum and the oval windows of the dome, which is reflected by the tensile stresses identified by the model in this part of the structure. The direction of the cracks that spread from the oval windows up into the dome is in good agreement with the pattern of tangential stresses identified in the vault. The diagonal cracks in the spurs are due to the horizontal actions of the dome: in the model they are identified as tensile stresses at the interface between the drum and the buttresses. The analysis proves also that the position of the original annular strengthening system is the most appropriate, and that the modern additional strengthening system located in the upper part of the drum is also in an efficient position. The effects of the settlements will be studied by a further step of the analysis extending the numerical model to incorporate the underlying soil, after a geotechnical characterization of the foundation layers.

5. Reference list

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